

Characterization of smooth symbol classes by Gabor matrix decay

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We introduce the symbol classes S^m , $m \in \mathbb{R}$, consisting of smooth functions σ on \mathbb{R}^{2d} such that $|\partial^\alpha \sigma(z)| \leq C_\alpha (1 + |z|^2)^{m/2}$, $z \in \mathbb{R}^{2d}$; the Hörmander class $S_{0,0}^0$ is recaptured for $m = 0$. We show that they can be characterized by an intersection of different types of modulation spaces. We exhibit almost diagonalization properties for the Gabor matrix of τ -pseudodifferential operators with symbols in such classes, extending the characterization proved by Gröchenig and Rzesotnik. Finally, the Gabor matrix of a Born-Jordan operator is computed and new boundedness results for such operators are inferred.